The Laloy & Massard model: improvements and limitations – *How do numerics help us in making better and useful experiments*

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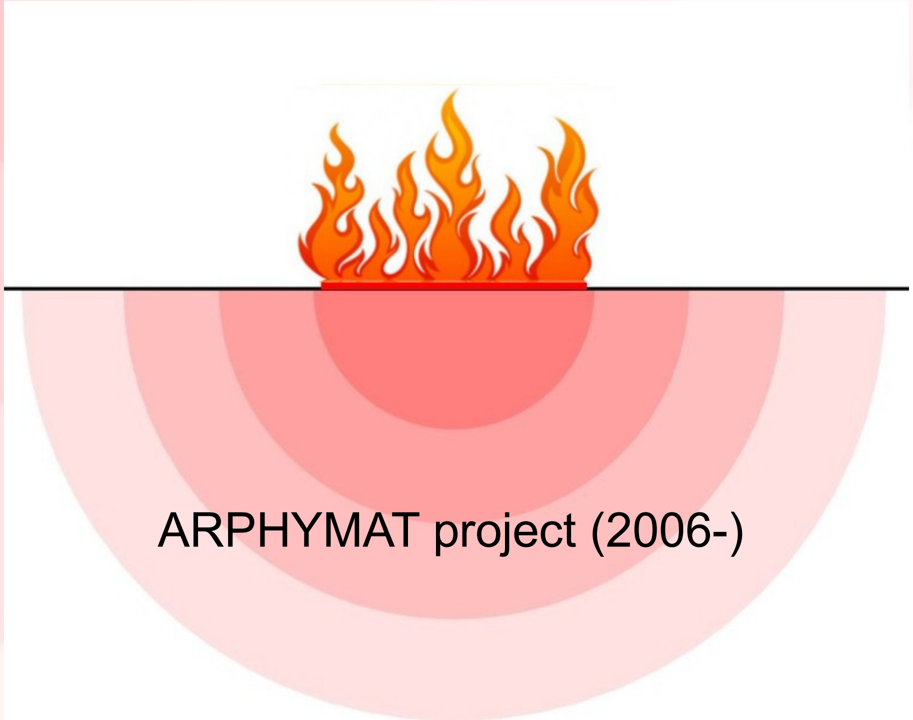


ARPHYMAT project (2006-)

The heat equation

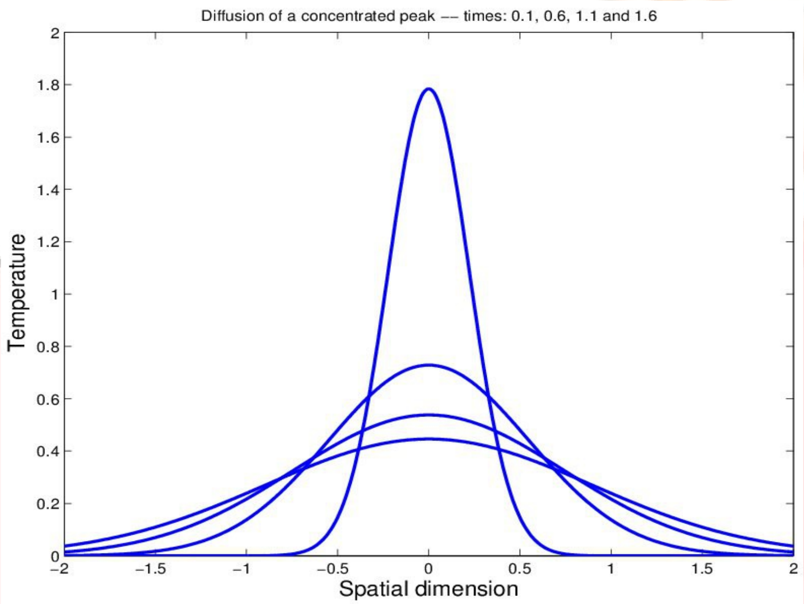
$$\boxed{\rho C_p \frac{\partial T}{\partial t}} + V.\nabla T = \boxed{\mathrm{div}\left(\lambda(\mathbf{x})\nabla T\right)} + q$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\alpha = \frac{\lambda}{\rho C_p}$$

Diffusion equation: same for heat diffusion, flow in porous media (Darcy) or gas diffusion ...

Diffusion of a peak of temperature



L

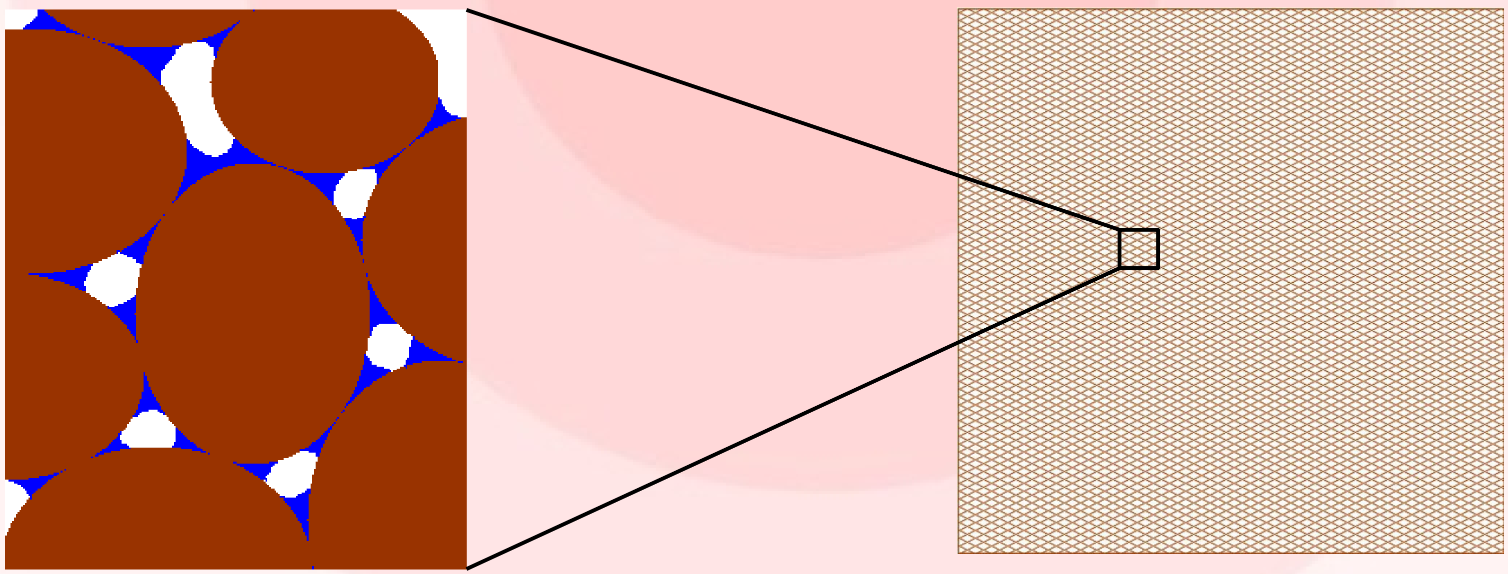
Gaussian shape (*i.e.* exponential) is typical for elementary solutions.

Diffusion process is more and more slower:

$$\tau_c = \frac{L^2}{\alpha}$$

Homogeneous / non-homogeneous material

- Homogeneous material: λ = cst
- Non-homogeneous: $\lambda \rightarrow$ scalar function of the position
- Non isotropic behavior: λ is a diagonal matrix



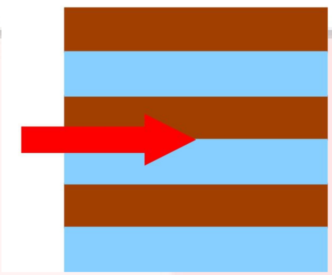
micro-scale:
non-homogeneous $\lambda(x)$

macro-scale:
homogeneous λ_e

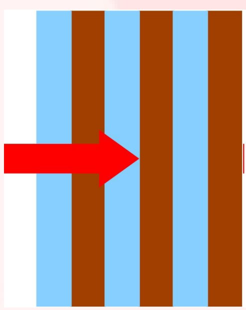
→ need of effective values in simulations

Effective properties?

- easy to compute for density (ρ) and heat capacity (C_p) : extensive

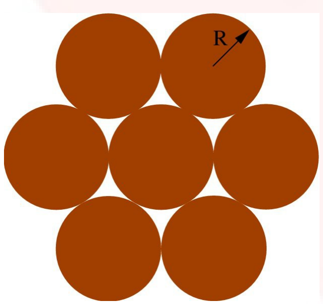


λ_e : arithmetic mean



λ_e : harmonic mean

General case:
experimental measurement
(not so easy!)

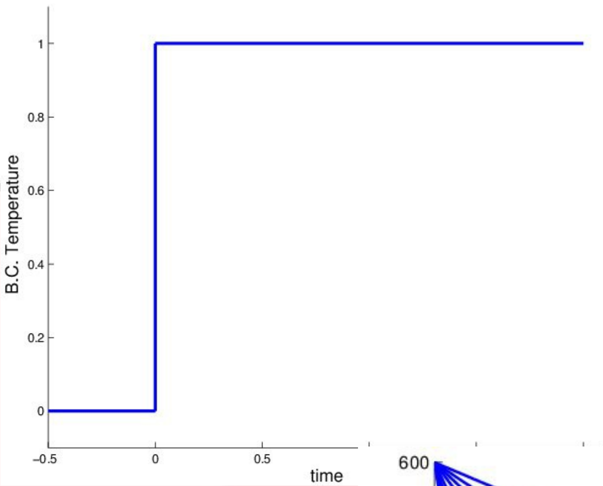


λ_e : some models available
(*e.g.* Kunii & Smith, 1960, for compact spheres)

Analytical solutions: two different B.Cs.

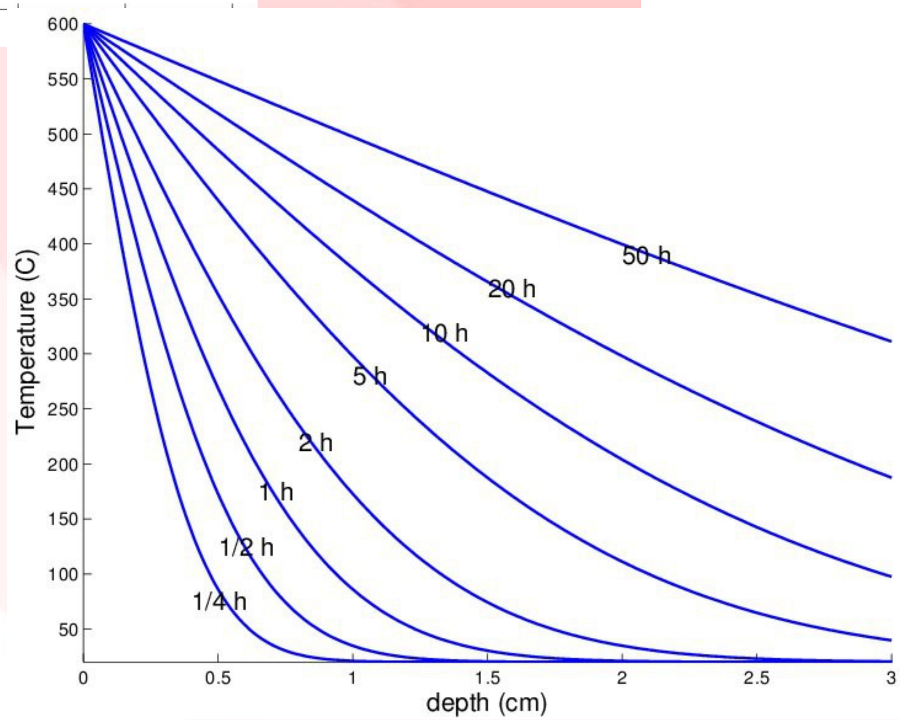
sudden heating

(close to the archaeological fire)



$$T(z,t) = T_0 + (T_1 - T_0)\ \mathrm{erf}\left(\frac{z}{2\sqrt{\alpha t}}\right)$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2)\,du$$

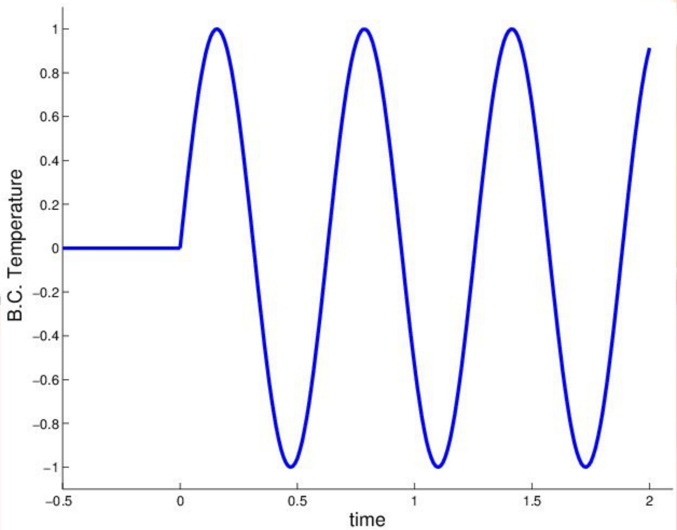


$T(z,t)$ = transient part $\rightarrow 0$

+

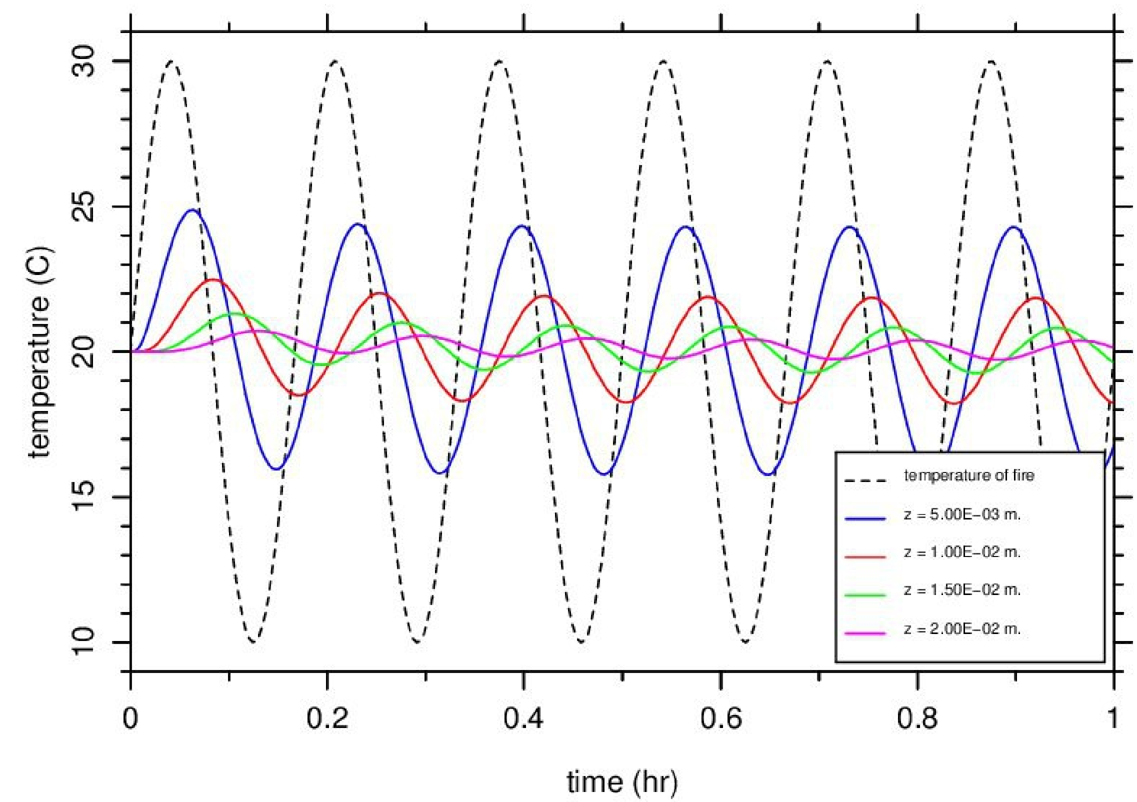
established part (periodic)

periodic heating



$$T(z,t) \rightarrow T_0 + A\exp(Bz)\,cos(\omega\,t + Bz)$$

$$B = \sqrt{\frac{\omega}{2\alpha}}$$



Laloy & Massard method (1)

- Laloy & Massard (1984) → archaeological fires
- find diffusivity (α) by measuring transient temperatures only (sudden heating)
- isotherms curves must be plane (1D solution)
- semi-infinite medium
- easy to apply (excel sheet) but *approximation*

Laloy & Massard method (2)

$$\exp(-2x^2) \leq \mathrm{erf}(x)^2 \leq \exp(-x^2)$$

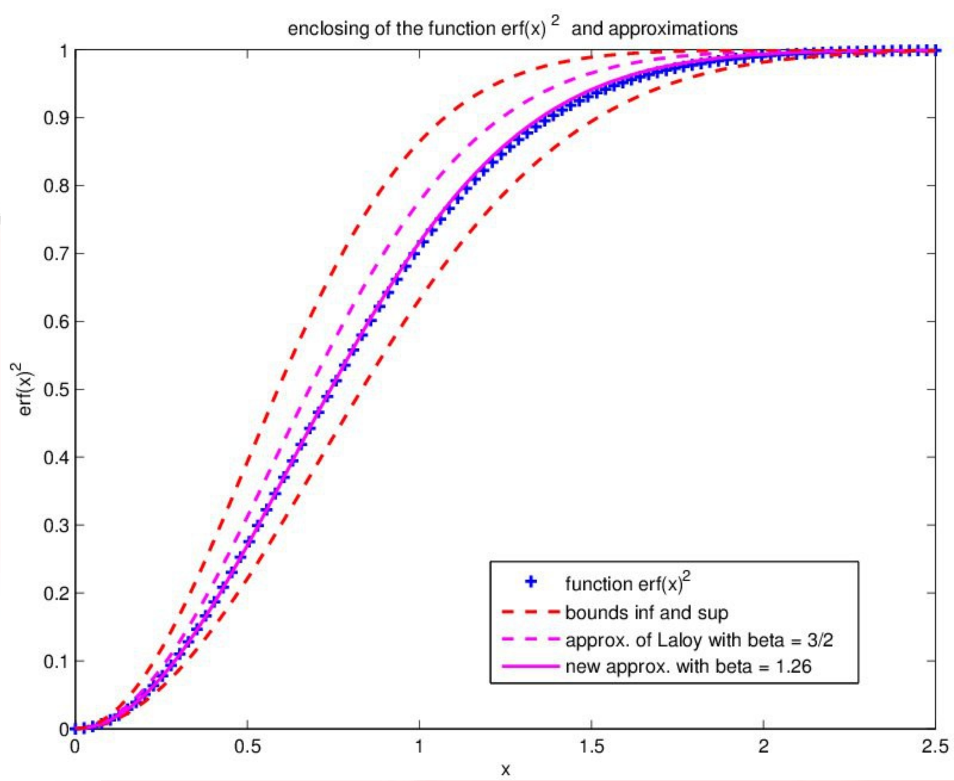
- Laloy and Massard initial approximation:

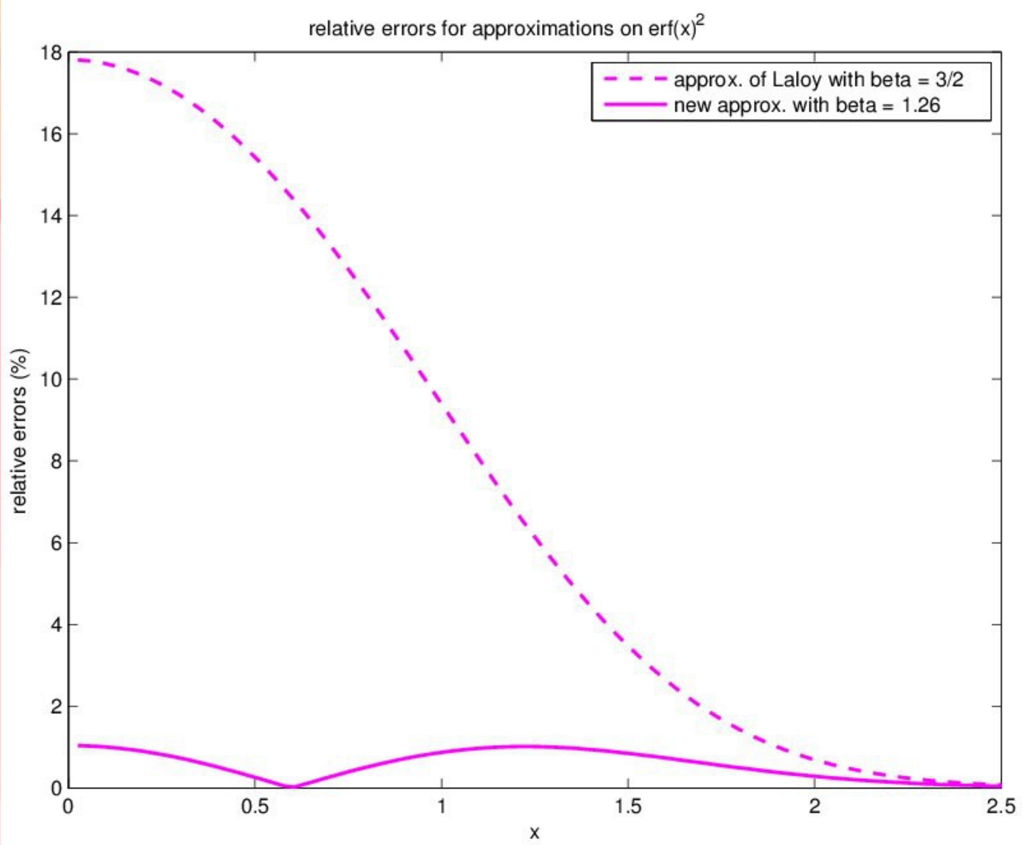
$$\mathrm{erf}(x)^2 \approx \exp(-\frac{3}{2}x^2)$$

But this can be greatly improved: (see results)

$$\mathrm{erf}(x)^2 \approx \exp(-1.26x^2)$$

Laloy & Massard method (3)





Initially, synthetic data (from numerical simulation) : 18 % relative error

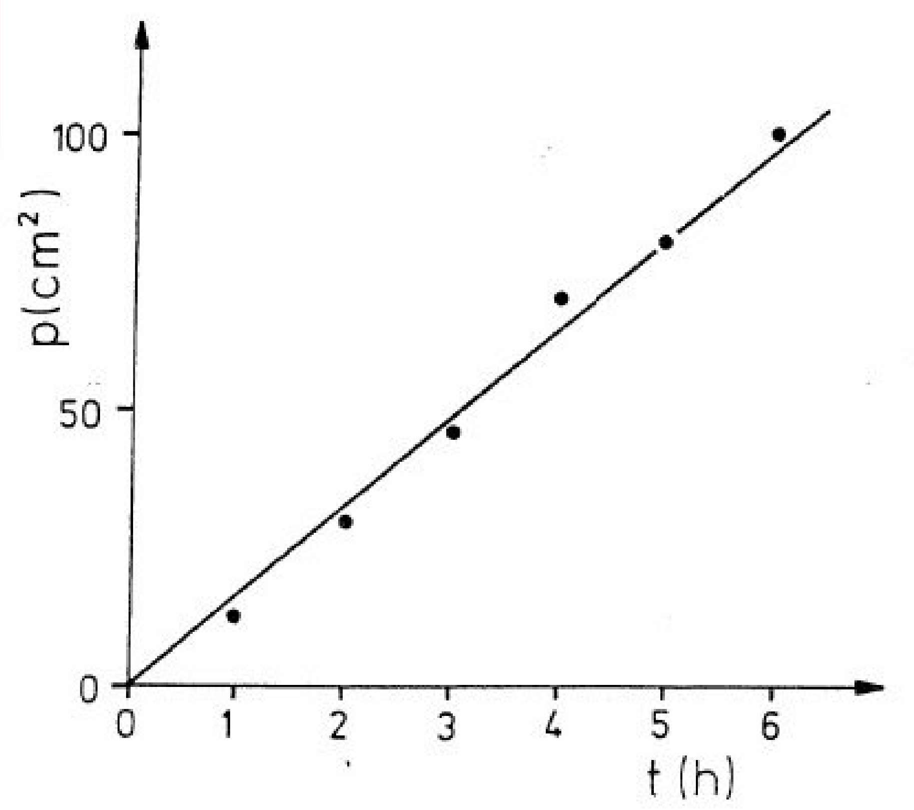
New math. Approx → 1 % relative error

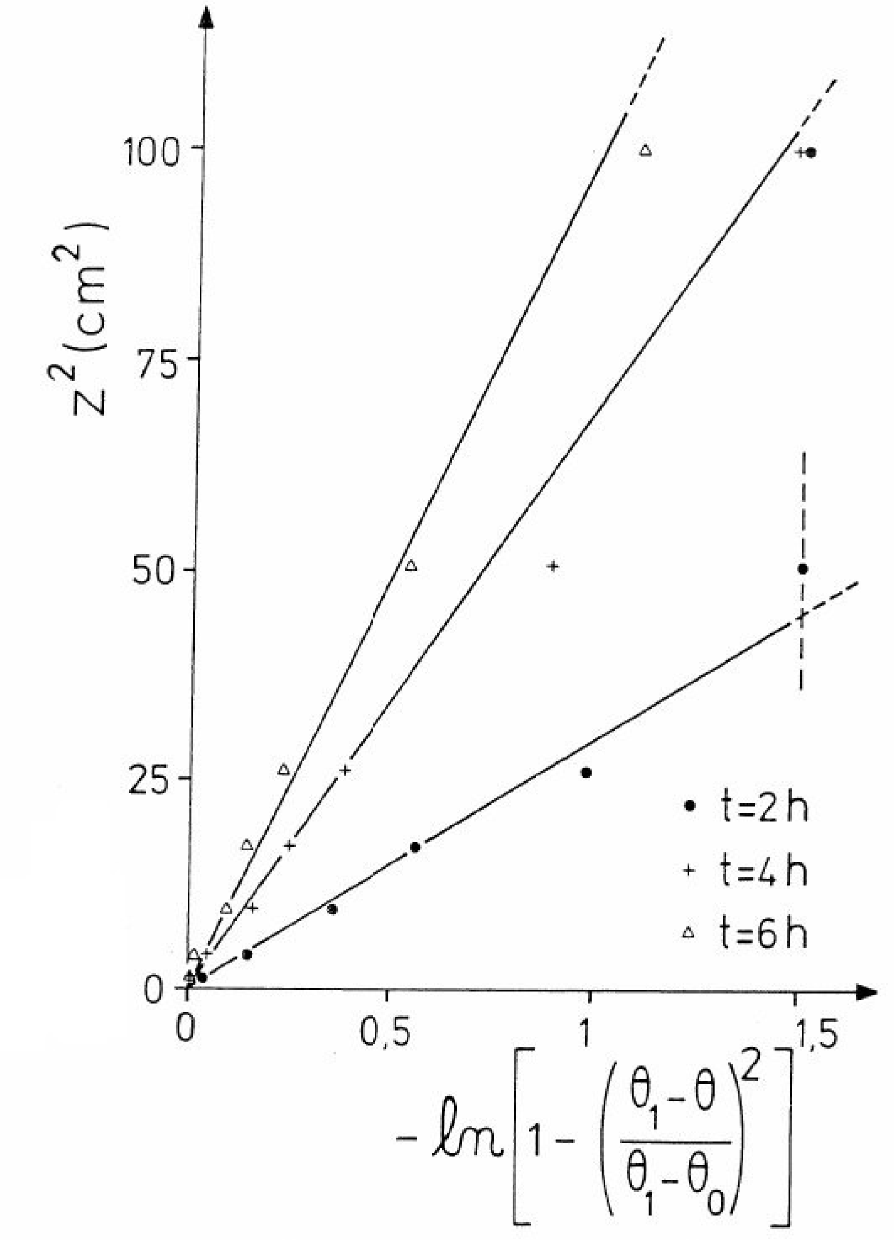
Laloy & Massard method (4)

$$\log\left[1-\left(\frac{T_1-T(z,t)}{T_1-T_0}\right)^2\right]=-\beta\frac{z^2}{4\alpha t}$$

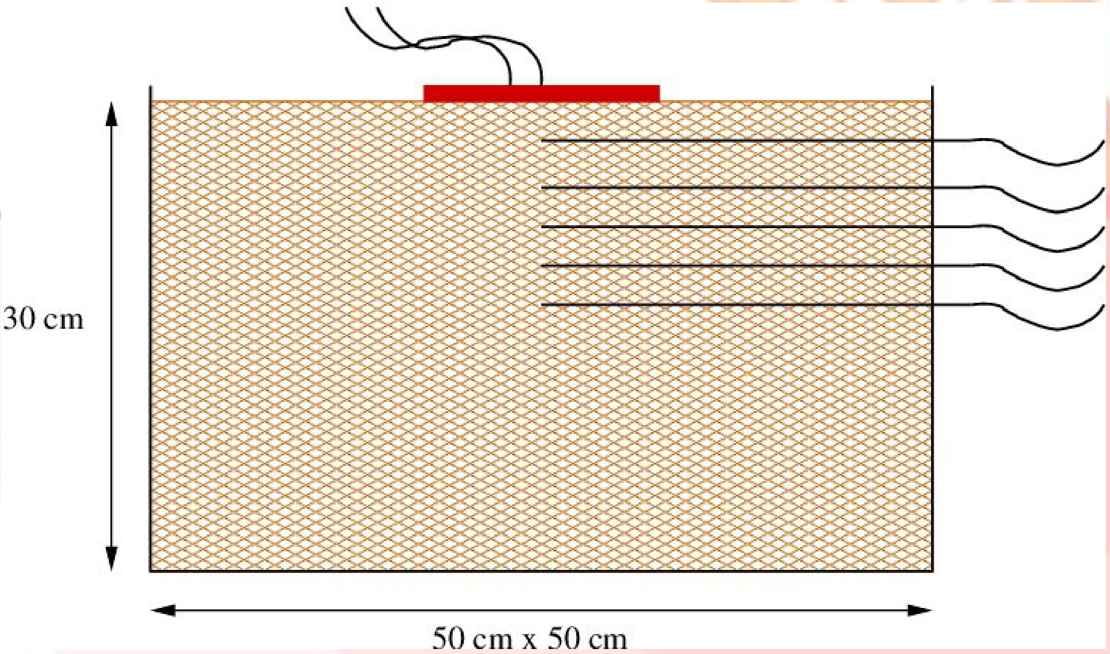
$\beta = 1.26$

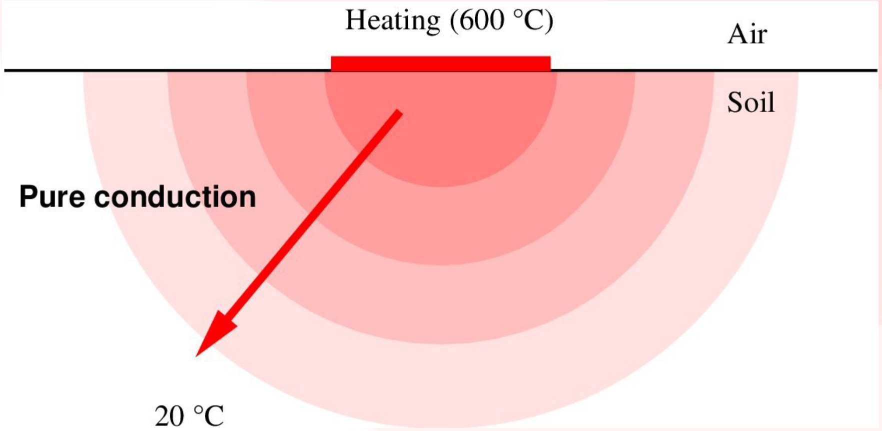
slope p prop. to $(4/\beta)\,\alpha\, t$ →

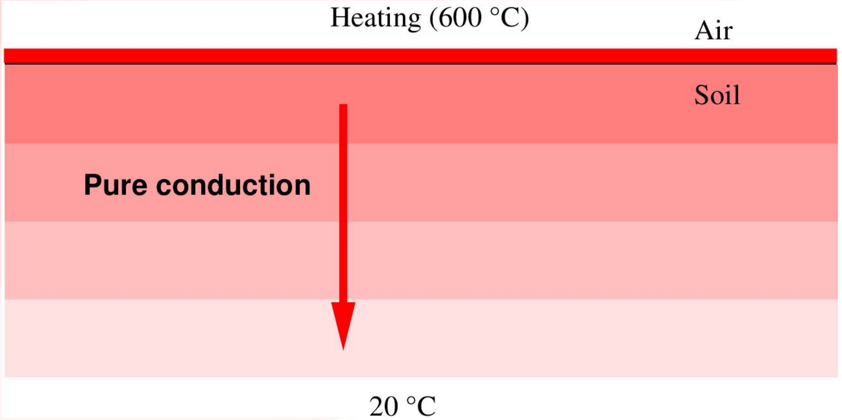




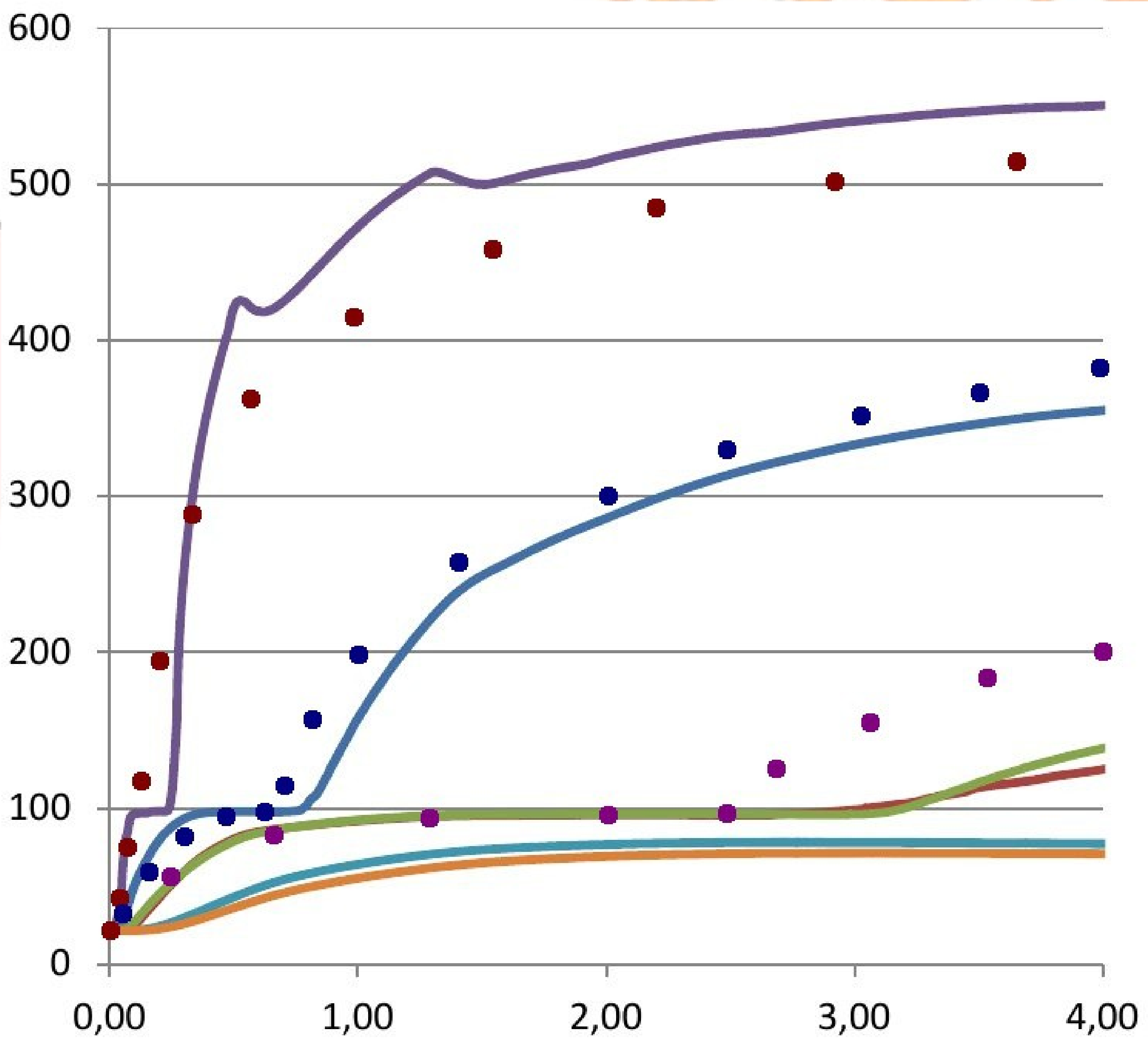
Application to ARPHYMAT







Comparison between experiments and simulations



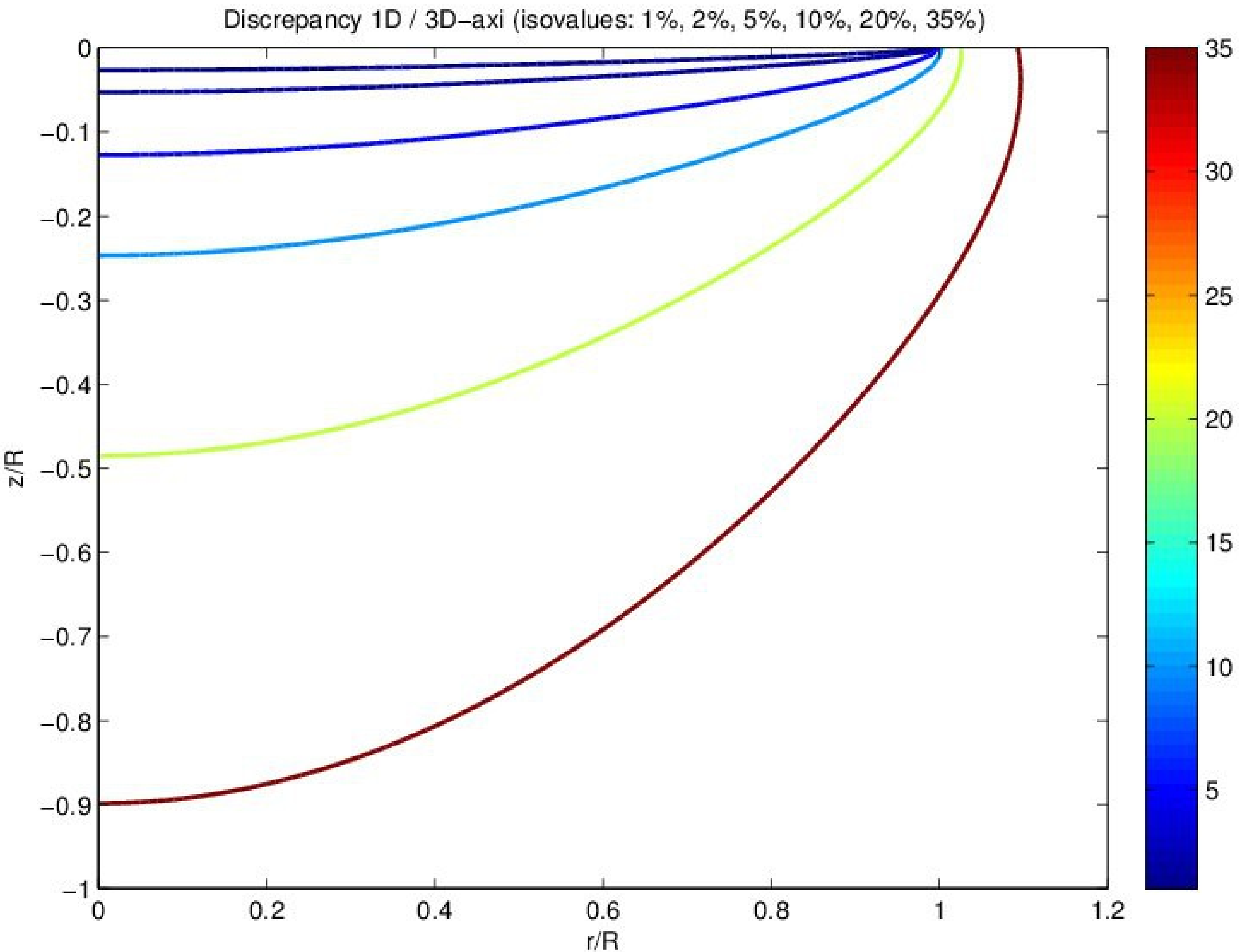
even for dry sand, comparison is not good!

→ looking for better diffusivity values (α)

Errors' sources in applying the Laloy & Massard method

- isotherms not parallel in the big setup
- even for the small setup, sides are not well isolated
- temperature of the plate (B.C.) is not sudden
  (must wait 10 to 20 minutes to reach the imposed temperature)
- boxes have finite depth (side effect)

Validity of the 1D approx. for the big setup



New method for the determination of α

- periodic B.C. (z=0) → $T_0(t)$
- at least one sensor (*e.g.* $z=z_1$) → $T_1(t)$
- use Discrete Fourier Transform (on both signals)
- get amplitude A(f) = 2 abs(c) → max. amplitude A_0 and A_1

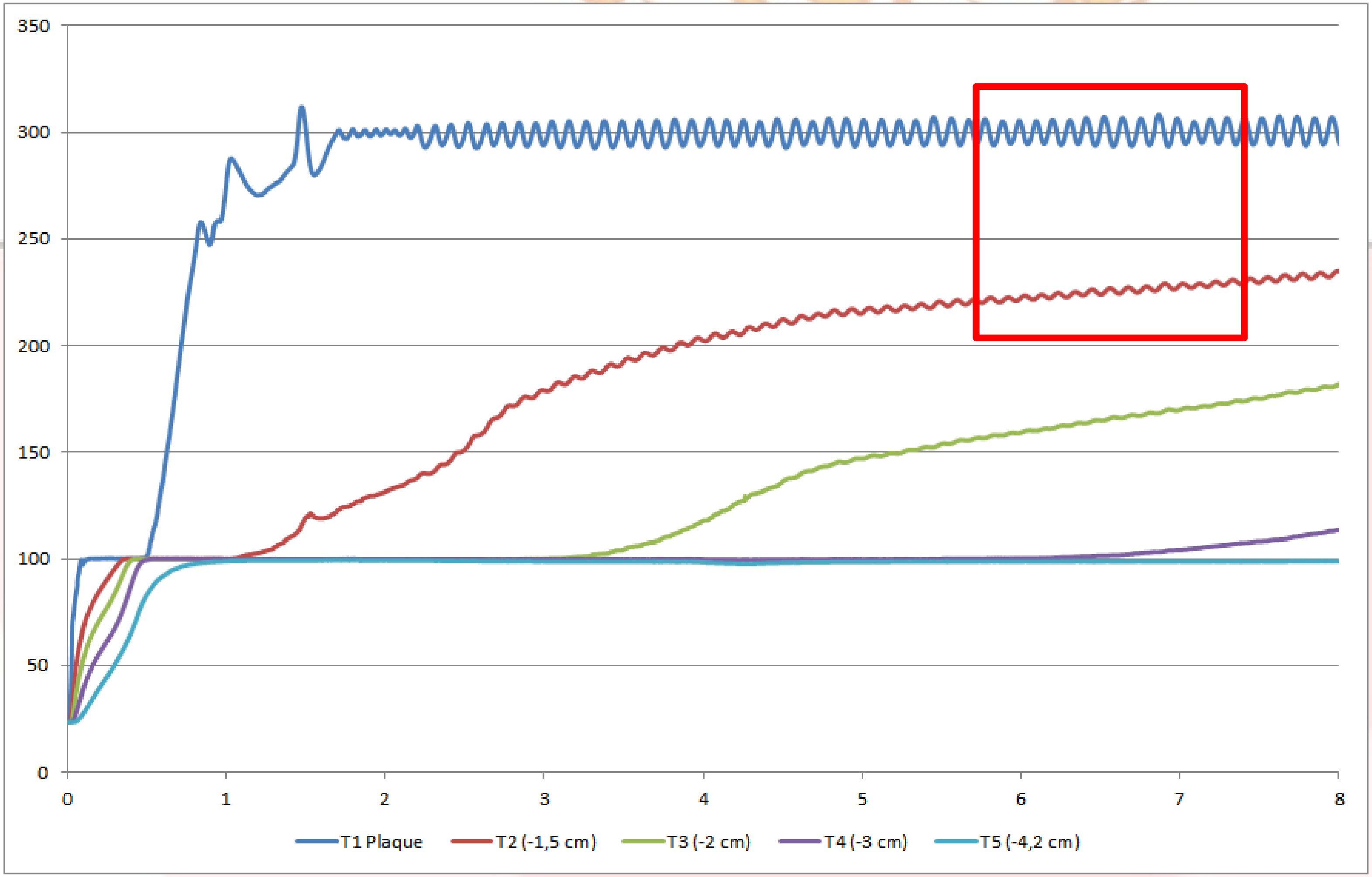
$$\frac{A_1}{A_0} = \exp\left[\sqrt{\frac{\omega}{2\alpha}}(z_1 - z_0)\right]$$

Advantages of this new method

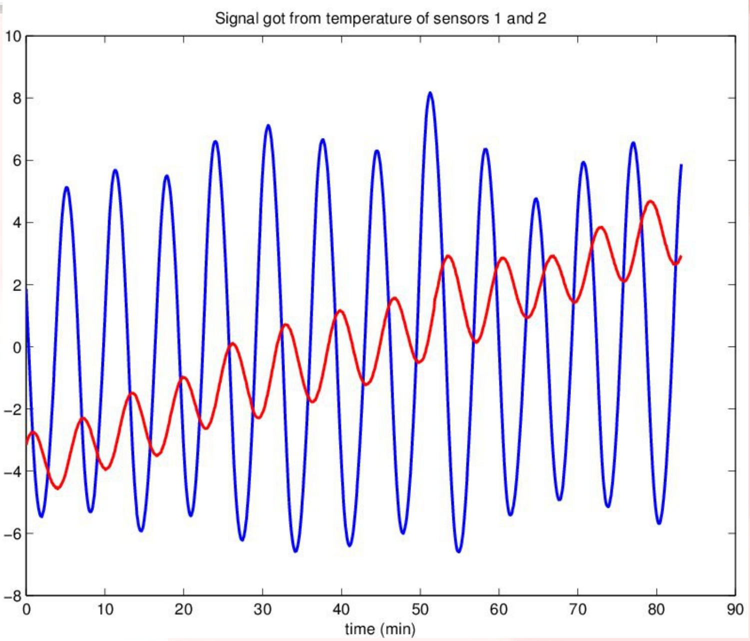
- repeated experiments for different values of T0 $\rightarrow$
  α = function(T)
  i.e. diffusivity depends on temperature

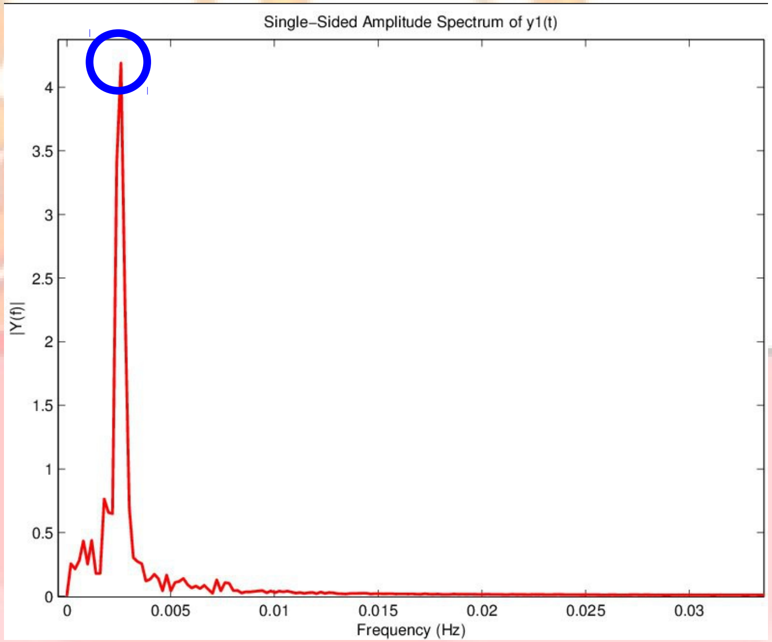
- choice of pulsation ω: optimal way?
  - small value of $\omega \rightarrow$ better precision in T
  - large value of $\omega \rightarrow$ avoid side effect at the bottom
- no need to reach the established region, thanks to the linearity of the heat equation (see next slide)

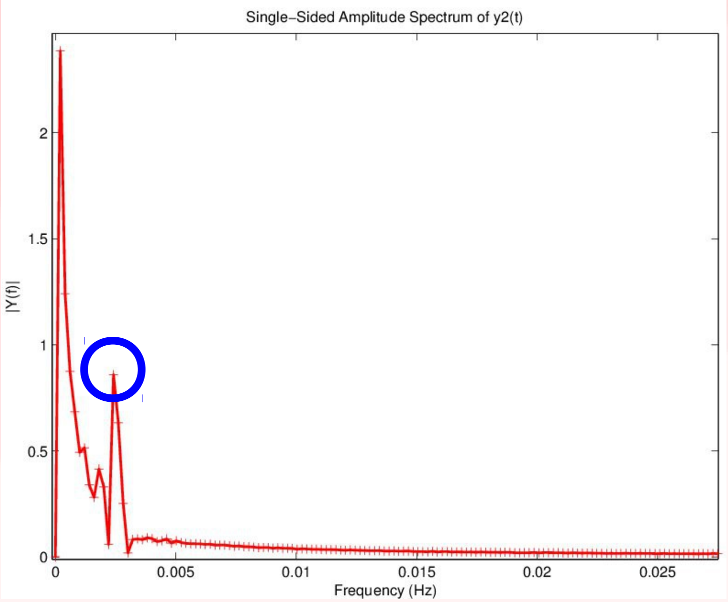
Example of application



Example of application







$\alpha \sim 7.3\ 10^{-7}\ m^2/s$

Conclusion and Perspectives

- deep knowledge of physical processes is required to understand the difficulties in experimental methods. (making good experiments is hard)
- numerical computations lighten the whole process, validate some assumptions and predict the order of experimental uncertainties.
- analytical solutions (*i.e.* exact mathematical solutions) are always a “plus” in deriving a new method.
- we plan to use the new periodic B.C. in the determination of diffusivity.
- we seek an analytical, closed form, solution for the 3D-axi, in oblate spheroidal coordinates, for the periodic B.C. case.